Explicit Equations for Estimating Resistance to Flow in Open Channel with Movable Bed Based on Artificial Neural Networks Procedure

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ABSTRACT (10PT)

The resistance to flow in an open channel is associated with the value of the Darcy-Weisbach friction factor *f*. For natural channels with a movable bed, the *f* value depends on the grain size of the bed materials and the bedforms, such as ripple, dune, or anti-dune. The total resistance to flow is the sum of the resistance due to grain roughness and bedform. Several researchers have proposed several graphs to determine the friction factor value due to the bedforms. Still, using these graphs requires graphical interpolation, which is inconvenient and difficult to apply to the flow and sediment transport calculation. This study proposes two explicit equations, ANN models 1 and 2, to compute the friction factor due to the bedform based on artificial neural networks (ANN) procedure. The data used to build the equations were obtained by digitizing the graph proposed by Alan and Kennedy. The explicit ANN equations are in the form of a series of hyperbolic tangent functions. The resulting equations can predict the friction factor value due to bedform satisfactorily.

I. Introduction

The resistance to flow in an open channel is related to the magnitude of the friction factor f, which is defined by equation (1) [1-4].

$$f = \frac{8gRS}{V^2} \quad (1)$$

Where R = hydraulic radius of the wet section, S = energy slope, V = average flow velocity, and g = acceleration due to gravity. From Equation (1), one can have equations (2) and (3)

$$V = \sqrt{\frac{8g}{f}}\sqrt{RS} \quad = C \quad \sqrt{RS} \quad (2)$$

$$C = \sqrt{\frac{8g}{f}} \quad (3)$$

where *C* = Chezy coefficient. Friction factor *f* and Chezy coefficient *C* depend on hydraulically smooth or roughly turbulent flow conditions. Flow in natural channels, such as rivers, is generally turbulent, and resistance to flow is often related to the Manning coefficient, *n* [1-4]. The relationship between the average velocity and the Manning coefficient *n* is expressed by equation (4)

$$V = \frac{1}{n} R^{2/3} S^{1/2} \tag{4}$$

From equations (2) and (4), the relationship between Chezy *C* and Manning coefficient *n* is obtained as stated by equation (5)

$$C = \frac{R^{1/6}}{n} \quad or \;\; n = \frac{R^{1/6}}{C} \tag{5}$$

From equations (3) and (5), the relationship between the friction factor *f* and the Manning coefficient *n* can be obtained as given by equation (6).

$$f = \frac{8gn^2}{R^{1/3}} \tag{6}$$

For channels with a plane bed flow, the value of *C* or *n* is constant and is related to the grain size of the bed materials. The value of *n* is given in some literature [1-4,7]. However, for open channel flows with a planned bed consisting of sand and gravel, the Manning coefficient *n* is related to the grain size of the bed material, as stated by equation (7) [5,7].

$$n = \frac{d^{1/6}}{A_n} \tag{7}$$

Where d = the diameter of the bed material. For $d = d_{50}$, the value $A_n = 20$ (see Wu [5]). In natural channels, the bed material can undergo a process of erosion and sedimentation so that it is not fixed. The bedform can form patterns like ripple, dune, and antidune [5-7]. These bedforms will increase the resistance to flow—resistance which is a combination of grain roughness and bedform [5-7]. Thus, the friction factor *f* in an open channel with a movable bed is expressed in equation (8) [7].

$$f = f' + f'' \tag{8}$$

Where f' and f'' are the friction factor due to grain and bedforms, respectively, and *f* is total friction factor [7]. The value of f' can be determined by equation (7) [5], but the value of f'' depends on flow characteristics and sediment parameters. The value of f'' is generally determined through experiments in the laboratory and observations in the field. Alan and Kennedy (see Yang [7]) produced a graph to determine the value of f''. They propose a relationship as in equation (9)

$$f'' = F\left(\frac{V}{(gd_{50})^{1/2}}, \frac{R}{d_{50}}\right) \tag{9}$$

They produce a graph of the functional relationship Equation (9). However, the resulting graph requires graphical interpolation to get the f'' value, making it less convenient and cumbersome for practical implementation. Moreover, it isn't easy to include it in hydraulic and sediment transport computations.

This study proposes an explicit functional relationship Equation (9) using the artificial neural networks (ANN) procedure. The explicit equation determines the value of f'' as a function of the parameters $X = V/(gd_{50})^{1/2}$ and $Y = R/d_{50}$. The data for constructing the ANN models was obtained by digitizing the graph provided by Alan and Kennedy given in Yang [7]. The resulting equation can be applied to calculate the friction factor in a channel with a movable bed. Furthermore, the following sections describe the structure of the ANN, the method of developing explicit ANN equations, data collection for training, validation and testing, analysis, and conclusions.

II. Methods

A. Development of Explicit ANN

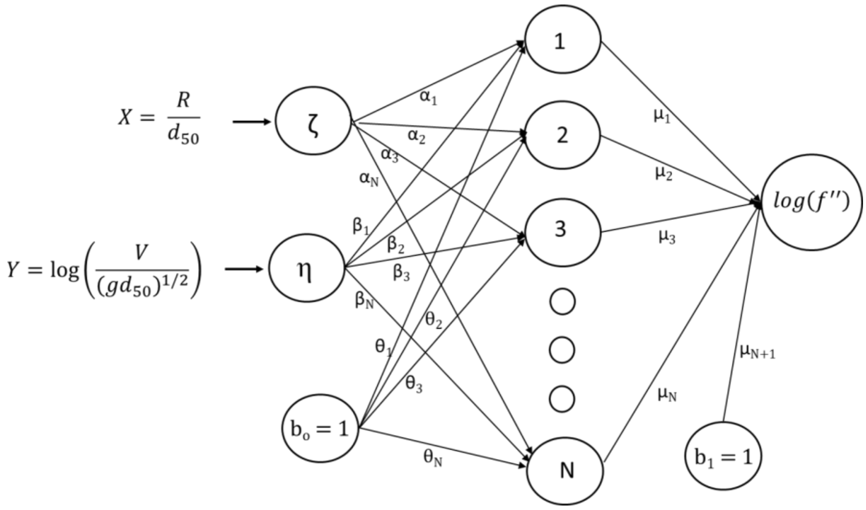


Fig. 1. Configuration ANN used

The input parameters used are the logarithmic values of $X = R/d_{50}$and $Y = V/(gd_{50})^{1/2}$Y, while the output parameter is the logarithmic value of f''. In this study, a Multi-layer Perceptron (MLP) structure is used [8-10], consisting of input, hidden, and output layers. Two ANN models are proposed, namely models 1 and 2, using five and ten neurons in the hidden layer. Figure 1 shows a configuration of the ANN structure used. The explicit ANN equations are based on the equation developed by Cahyono [12-13]. Thus, the explicit ANN for models 1 and 2 with the structure in Figure 1 can be expressed by equations (10) and (11).

$$\log(f'') = \sum_{i=1}^{N} B_i \, tanh(\alpha_i\zeta + \beta_i\,\eta + \theta_i) + C \tag{10}$$

$$\log(f'') = A_1 tanh(\alpha_1\zeta + \beta_1\eta + \theta_1) + A_2 tanh(\alpha_2\zeta + \beta_2\eta + \theta_2) + A_3\, tanh(\alpha_3\zeta + \beta_3\,\eta + \theta_3) + \cdots + A_N\, tanh(\alpha_N\zeta + \beta_N\,\eta + \theta_N) + B \tag{11}$$

Where ζ and η are normalized input parameters X and Y, respectively, α_i, β_i, θ_i μ_i, A_i and B are coefficients. The values of coefficient α_i, β_i, θ_i and μ_i in Fig. 1 (i = 1, 2,.., N) and $\mu_{N+1,}$ and also coefficient A_i and B are determined by optimization technique with the objective function to minimize the sum of squared errors defined by Equation (12) in the training process.

$$min\ e = min \sum_{i=1}^{M} \left(f''_{A,i} - f''_{E,i}\right)^2 \tag{12}$$

where $f''_{A,i}$ and $f''_{E,i}$ are the desired value of f''computed by explicit AAN model given by Equation (10) and (11), and data obtained by digitizing the chart, respectively.

B. Statistical Measure

The performance of models is assessed using the following statistical measures:

1. The coefficient of determination, R^2, of the linear regression line between the $f''_{A,i}$ and $f''_{E,i}$
2. The mean absolute relative error, *MRE,* defined by:

$$MRE = \frac{1}{M} \sum_{i=1}^{M} \left|\frac{f''_{E,i} - f''_{A,i}}{f''_{E,i}}\right| x\ 100 \tag{13}$$

Where $RE = \left|\frac{f''_{E,i} - f''_{A,i}}{f''_{E,i}}\right|$ is absolute relative error and M is amount of data.

3. The maximum absolute relative error, MAXRE, defined by:

$$MAXRE = max\left|\frac{f''_{E,i} - f''_{A,i}}{f''_{E,i}}\right| x\ 100\ \%\ , for\ \ i = 1, 2, \ldots., M \quad (14)$$

III. Results and Discussion

The data for training, validation and testing of the models are obtained by digitizing the charts provided by Alan and Kennedy given in Yang [7] using WebPlotDigitizer software (https://automeris.io/WebPlotDigitizer/, accessed on 20 August 2021). The digitization process produces 472 data points with X= $V/(gd_{50})^{1/2}$ from 5 to 35 and $Y = R/d_{50}$ from 140 to 40000. From these digitized data, 331 were used for training and 141 for validation and testing. The training using the MATLAB 2022a program provides coefficients α_i, β_i, θ_i and μ_i for both ANN models 1 and 2. These coefficient values are used as the initial condition for further optimizations in the MS Excel program developed by author. These optimizations are conducted using the objective function of minimizing MAXRE as defined in Equation (14) to further minimize the relative error of the predicted f''. The optimization results produce the coefficients α_i, β_i, θ_i, A_i and B for the model 1 given in Table 1 and the corresponding values for the ANN model 2 are shown in Table 2. Thus, the explicit formula for ANN model 1 is given by equation (15) below.

$$\log(f'') = A_1 tanh(\alpha_1\zeta + \beta_1\eta + \theta_1) + A_2 tanh(\alpha_2\zeta + \beta_2\eta + \theta_2) + A_3\, tanh(\alpha_3\zeta + \beta_3\,\eta + \theta_3) + A_4\, tanh(\alpha_4\zeta + \beta_4\,\eta + \theta_4) + A_5\, tanh(\alpha_5\zeta + \beta_5\,\eta + \theta_5) + B \quad (15)$$

Where $\zeta = 0.0667\left(V/(gd_{50})^{1/2} - 5\right) - 1$ and $\eta = 0.8225(\log(R/d_{50}) - 2.16) - 1$, with coeeficien α_i, β_i, θ_i , A_i and D are given in Table 1. The ANN model 2 is defined by Equation (16)

$$\log(f'') = A_1 tanh(\alpha_1\zeta + \beta_1\eta + \theta_1) + A_2 tanh(\alpha_2\zeta + \beta_2\eta + \theta_2) + A_3\, tanh(\alpha_3\zeta + \beta_3\,\eta + \theta_3) + A_4\, tanh(\alpha_4\zeta + \beta_4\,\eta + \theta_4) + A_5\, tanh(\alpha_5\zeta + \beta_5\,\eta + \theta_5) + \cdots + A_{10}\, tanh(\alpha_{10}\zeta + \beta_{10}\,\eta + \theta_{10}) + B \quad (16)$$

Where ζ and η as defined in Equation (15) and with coefficients α_i, β_i, θ_i , A_i and D are given in Table 2.

Table 1. Coefficients of explicit ANN model 1 defined by Equation (15)

	Coefficient				
i	α_i	β_i	θ_i	A_i	B
1	-3.6517	8.0011	4.2222	0.9623	-2.0872
2	3.4994	-0.7895	-2.2512	-0.1274	
3	0.8684	-0.1008	0.4790	-0.8600	
4	3.0195	5.4351	6.2120	1.5260	
5	-2.9787	-4.4793	-5.6210	1.7302	

Table 2. Coefficients of explicit ANN model 1 defined by Equation (15)

	Coefficient				
i	α_i	β_i	θ_i	A_i	B
1	9.9067	2.8338	-10.6335	-0.0933	-2.6321
2	11.0880	0.2176	-4.1546	-0.0574	
3	-1.3734	8.0810	6.4734	3.0706	
4	-4.2480	8.5534	4.2440	0.5910	
5	41.7648	-54.8613	-14.0334	-0.0414	
5	-28.8282	-16.9178	-35.1380	0.0698	

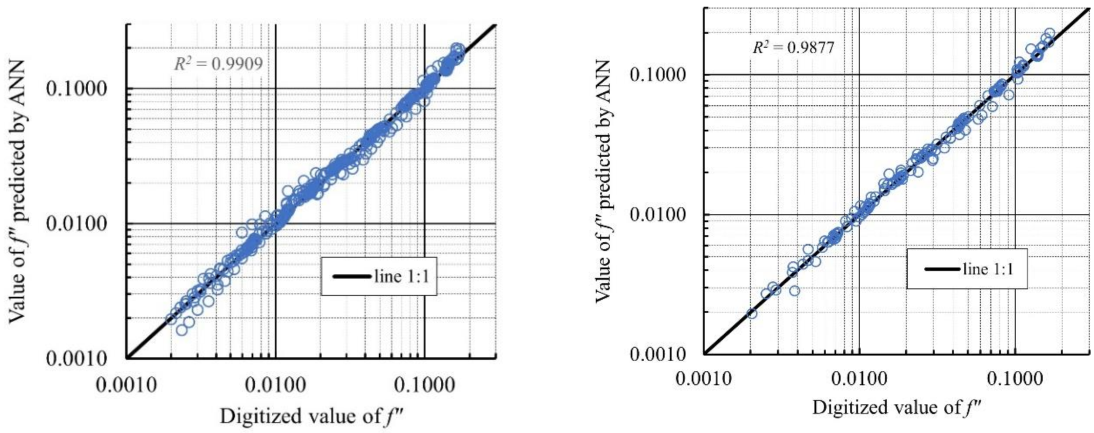
7	-1.8056	-9.1073	3.1834	0.0158
8	1.6909	2.9169	1.7905	-0.0718
9	1.0235	-0.2366	0.6529	-0.7198
10	-0.7727	9.6819	8.3334	-2.2663

The simulation results for the training, validation, and testing for the ANN models 1 and 2 produce the coefficient of determination R^2, *MRE* and *MAXRE* as shown in Table 3. It was discovered that both ANN models 1 and 2 produce satisfactory results in terms of R^2, *MRE* and *MAXRE* with the *MRE* observed to have varied from 1.1 % in training and to 1.6 % for training, validation, and testing.

Table 3. Statistical measures of models 1 and 2 for training and validation & testing data

ANN MODEL	Training			Validation and Testing		
	R^2	*MRE* (%)	*MAXRE* (%)	R^2	*MRE* (%)	*MAXRE* (%)
Model 1 (2-5-1)	0.9942	1.61	10.11	0.9877	1.59	10.44
Model 2 (2-10-1)	0.9974	1.10	7.88	0.9956	1.10	6.55

Figs. 2 and 3 show the scattering plot of the results of training as well as validation & testing for models 1 and 2, respectively. Simulations are also carried out for various X and Y values and the results are compared with the digitized data. Fig. 4 shows the curves obtained using the explicit equation (15) compared to the digitized data. The results for model 2 are given in Fig. 5. The results in Figs. 4 and 5 show that the curves generated by the ANN equations (15) and (16) almost pass through all the data points. Figs. 4 and 5 show that the expression ANN model (16) and 16) is very accurate in predicting the value of f''



(a) (b)

Fig. 2. Scattering plot of simulation results of model 1 for (a) training and (b) validation and testing data.

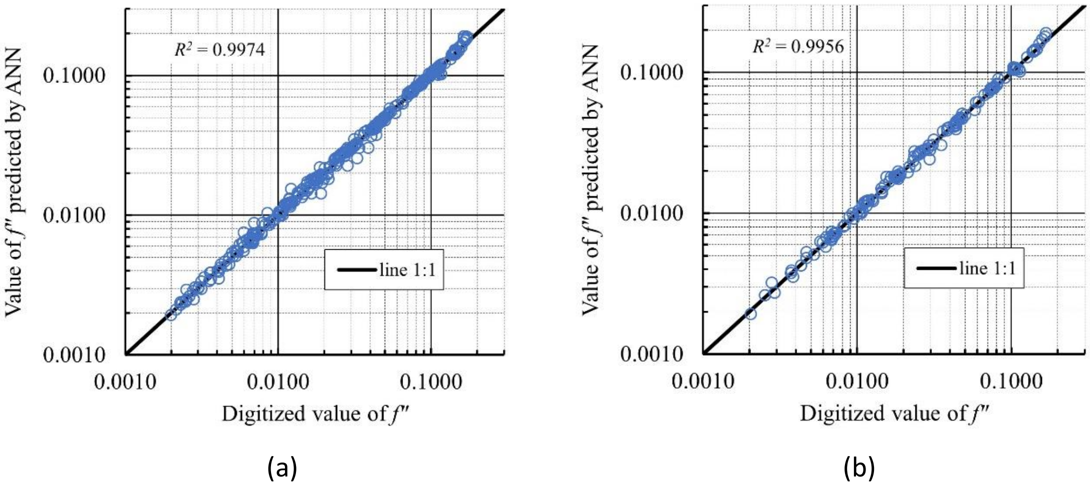


Fig. 3. Scattering plot of simulation results of model 2 for (a) training and (b) validation and testing data.

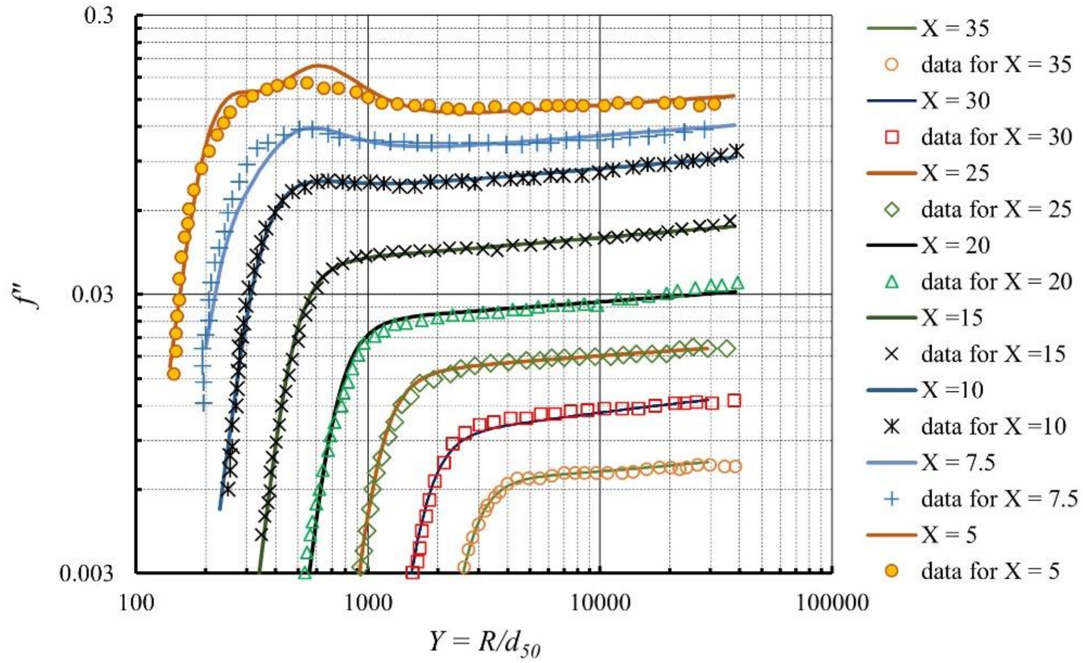


Fig. 4. Comparison between curves of the explicit ANN model 1 defined by Equation (15) and the digitized data for various X values.

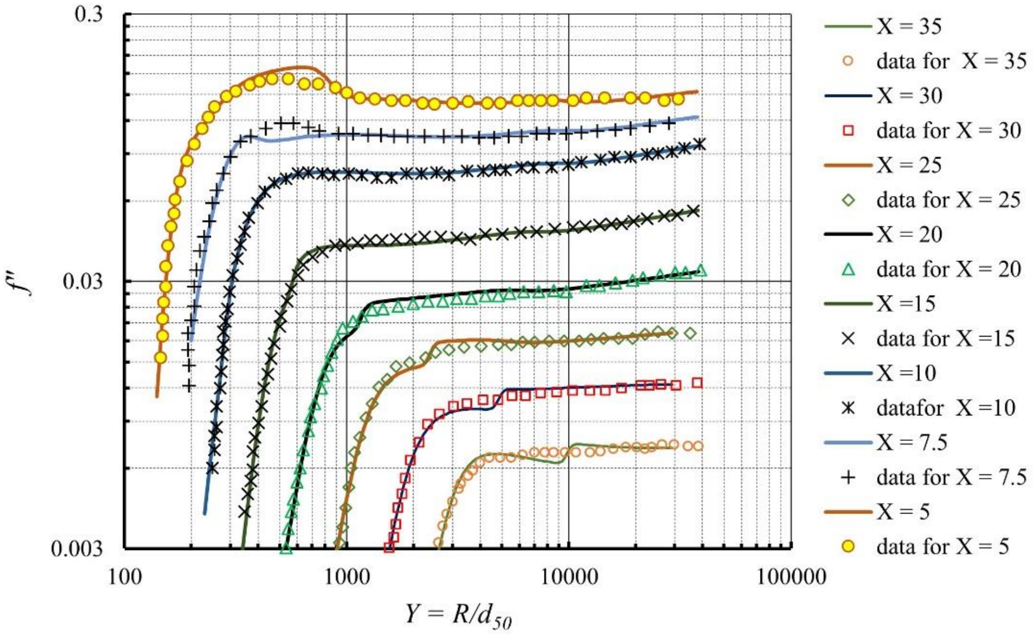


Figure 5. Comparison between curves of the explicit ANN model 2 defined by Equation (16) and the digitized data for various X values.

IV. Application of Explicit Equation

The following is an example of the implementation of an explicit ANN. Given an open channel with parameters d_{50} = 1 mm, R = 3 m, and S = 0.0002 and determine the flow velocity.

For fixed bed conditions, based on Equation (7), the Manning coefficient value is n' = (0.001)^1/6/20 = 0.0158 and the resulting flow velocity based on Equation (4) is V = 1.86 m/s. However, the flow velocity for the movable bed condition is obtained by iterating the following Equation (17).

$$V = C\sqrt{RS} = \sqrt{\frac{8g}{f'+f''}}\ \sqrt{RS} \qquad 17)$$

The value of f'' is determined using equation (15) or (16). Equation (15) or (16) depends on V, so Equation (17) is an implicit equation in V. Iteration solution using the Secant method [14] gives a value of V = 0.589 m/s. It is seen that the bedform formation will increase the resistance to flow, which in turn reduces the flow velocity.

V. Conclusion

This study proposes two explicit equations, models 1 and 2, to determine the friction factor f in an open channel with a movable bed. The equation is derived using the ANN procedure. The data used to build the ANN model is taken from the chart proposed by Alan and Kennedy given by Yang [7] through digitization. The proposed explicit ANN equation is a series function, namely the value of log(f'') as a hyperbolic tangent function of the parameter $\boldsymbol{\zeta = 0.0667\left(V/(gd_{50})^{1/2} - 5\right) - 1}$ and $\boldsymbol{\eta = 0.8225(\log(R/d_{50}) - 2.16) - 1}$ .The number of functions for models 1 and 2 is five and ten. The proposed ANN models can accurately predict the friction factor value due to bedform $\boldsymbol{f''}$ compared to digitized data. The proposed explicit ANN models can be easily combined in the flow calculation through iteration using the Secant method. The calculation example shows that the bedform effect can increase the resistance to flow and reduce the flow velocity significantly.

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